Modeling of an Asymmetric Turbulent Near Wake Using the Interaction Hypothesis

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Abstract

A MATHEMATICAL model for a turbulent near wake has been developed within the framework of the interaction hypothesis. The near wake behind an airfoil is treated as a complex turbulent flow consisting of two neighboring shear layers with distinct but overlapping shear stress profiles of opposite signs. The present model utilizes the mean momentum and continuity equations together with two shear stress transport equations derived from the turbulent kinetic energy equation. By relating the shear stresses to the local turbulence quantities, closure for the governing system of equations is achieved without the use of an eddy viscosity or mixing length. The shear stress is therefore no longer required to vanish at the velocity extremum. The model has been compared to the symmetric wake data of Chevray and Kovasznay and the asymmetric cascade wake data of Raj and Lakshminarayana. The agreement between the data and theory is acceptable with the former case superior to the latter.

Contents

The present analysis of an asymmetric turbulent near wake is based on the interaction hypothesis first proposed by Bradshaw et al.[1] in conjunction with a study of duct flows. Within the context of this hypothesis, a near wake can be regarded as an interaction between two neighboring shear layers with distinct but possibly overlapping shear stress profiles of opposite signs (Fig. 1). If the interaction is such that the turbulence structure in each layer is essentially unaffected by the presence of the adjacent layer, a superposition of the two shear stress fields for the purpose of calculating the net Reynolds stress is then possible. Moreover, by relating the shear stresses to the local turbulent quantities, as has been suggested by Bradshaw et al.,[2] two shear stress transport equations can be derived from the turbulent kinetic energy equation. Closure for the governing system is therefore achieved without making use of the eddy viscosity concept.

The mathematical model itself consists of the momentum, continuity, and a modified form of the turbulent kinetic energy equation, i.e.,

$$U\frac{\partial U}{\partial x} + V\frac{\partial V}{\partial y} = U_\infty \frac{\mathrm{d}U_\infty}{\mathrm{d}x} + \frac{\partial \tau}{\partial y}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

and

$$U\frac{\partial}{\partial x}\left(\frac{\tau^\pm}{2a_1{}^\pm}\right) + V\frac{\partial}{\partial y}\left(\frac{\tau^\pm}{2a_1{}^\pm}\right) = \tau^\pm \frac{\partial U}{\partial y}$$

$$\mp \frac{\partial}{\partial y}\left(G^\pm \tau^\pm |\tau_m{}^\pm|^{1/2}\right) \mp \frac{\tau^\pm |\tau^\pm|^{1/2}}{L^\pm}$$

where U and V are the time-averaged velocities, τ is the shear stress, a_1, L and G are the empirical functions of Bradshaw et al.[2] and the superscripts $\pm$ denote the positive or negative shear stress fields of Fig. 1. According to the interaction hypothesis $\tau = \tau^+ + \tau^-$, where τ^+ is the dominant shear stress in the region of positive shear—nominally where $y > y_c$ and y_c denotes the wake centerline—and vice-versa for τ^-.

In addition to the previous equations, the initial and boundary conditions must be prescribed. These consist of

$$U = U(0,y)$$

$$V = V(0,y)$$

$$\left\{\begin{matrix} U \to U_{+\infty} \\ \tau^+ \to 0 \end{matrix}\right\} \text{ as } y \to +\infty$$

$$\left\{\begin{matrix} U \to U_{-\infty} \\ \tau^- \to 0 \end{matrix}\right\} \text{ as } y \to -\infty$$

$$\tau^+ = \left\{\begin{matrix} \tau^+(0,y), & y \geq 0 \\ 0, & y < 0 \end{matrix}\right\}$$

$$\tau^- = \left\{\begin{matrix} 0, & y > 0 \\ \tau^-(0,y), & y \leq 0 \end{matrix}\right\}$$

where the velocity profiles $U(0,y)$ and $V(0,y)$ and the shear stress profiles $\tau^+(0,y)$ and $\tau^-(0,y)$ are ideally supplied by an upstream boundary layer calculation or measured directly from experiments.

The governing equations subject to the preceding boundary conditions have been integrated using an implicit finite dif-

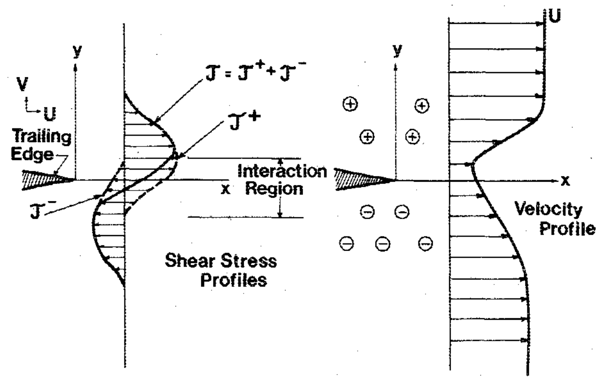

Fig. 1 The interaction hypothesis.

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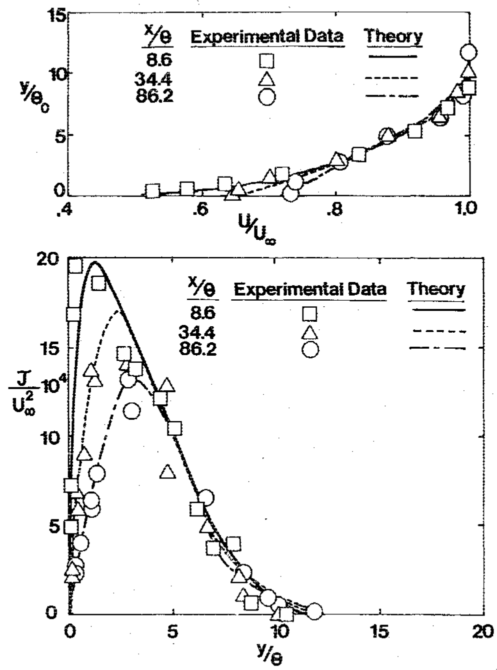

Fig. 2 Mean velocity and turbulent shear stress distribution in a symmetric wake.

ference technique developed by Ferriss[3] for turbulent boundary layers. The numerical scheme is believed to be quite stable but caution must be exercised to insure the accuracy of the solution. Although the effects of a Goldstein-type singularity at the trailing edge are expected to vanish quickly,[3] a recent study by Burggraf[4] suggests that an extremely fine mesh be used in the very near wake region. An x-step size of the order $R^{-3/5}$, where R is the chord length Reynolds number, must be used for the first 0.02-0.03 chord lengths downstream of the trailing edge.

The mathematical model has been compared to the measurements of Chevray and Kovasznay.[5] The data comparison is shown in Fig. 2. The current method accurately reproduces the experimental results and is superior to the Cebeci-Smith or Glushko techniques,[4] particularly with regard to the shear stress.

The interaction hypothesis has also been used to compute the asymmetric wake profiles behind a cascade of airfoils at an incidence of -6 deg and comparisons with the experimental measurements of Raj and Lakshminarayana[6] are shown in Figs. 3 and 4. The empirical functions are exactly those used in the symmetrical case. The inherent asymmetry is accounted for through the previously discussed scaling procedure.

The asymmetric nature of the initial conditions is such that the trajectory and/or locus of minimum velocity deviates from an extension of the mean airfoil camber line at the trailing edge. The velocity and shear stress profiles are plotted in terms of distances above and below this location. The overall agreement between the predicted velocity profiles and the data is good and the present results show considerable improvements over those obtained by a local similarity method,[7] particularly at the near wake stations.

Summary

Our investigation has validated the basic philosophy of the interactive approach for near wake calculations. However, it also clearly suggests that further refinements in the empirical functions based on reliable experimental data are needed before the present method can achieve the accuracy levels normally attributed to modern turbulent boundary layer calculations.

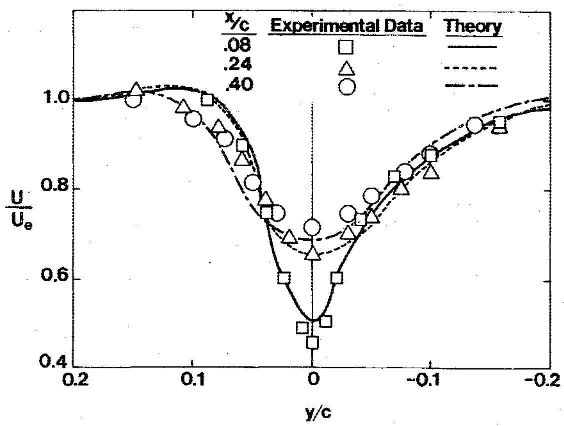

Fig. 3 Mean velocity distribution in an asymmetric wake.

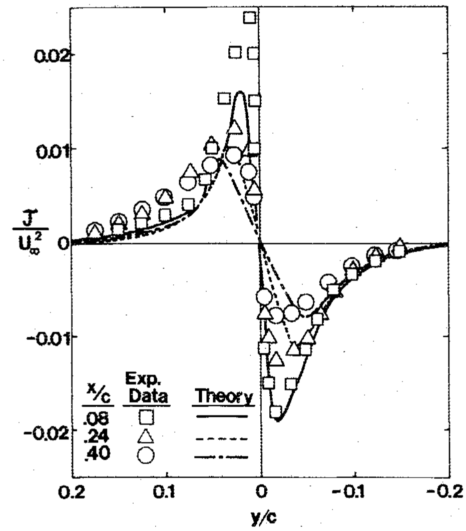

Fig. 4 Turbulent shear stress distribution in an asymmetric wake.

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